Numbers Uniquely Bias Spatial Attention: A Novel Paradigm for Understanding Spatial-Numerical Associations

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Abstract

Over the past two-and-a-half decades, numerous empirical studies have demonstrated a relationship between numbers and space. But what is the nature of this relationship? A classic interpretation is that these observed spatial-numerical associations (SNAs) are a product of a stable mental number line (MNL) in the mind. Yet others, concerned with task demands, have argued that SNAs are a product of transient mappings that occur in working memory. The difference is of great importance: whereas the latter interpretation has no implications for the representation of number, the former suggests that the representation of number is *inherently* spatial. Here, we tease apart questions of spatial representation (*à la* an MNL perspective) and spatial strategy (*à la* alternative accounts). In a novel place-the-number task, we demonstrate that numbers *automatically* bias spatial attention in a way that other ordinal sequences (i.e., letters) do not. We argue that this is evidence of an inherently spatial representation of number and explore how this work may help to answer future questions about the relationship between numbers and space.

Keywords: spatial-numerical associations (SNAs); mental number line (MNL); automaticity; working memory; polarity correspondence; synesthesia

Introduction

Since the seminal work of Dehaene, Bossini, and Giraux (1993), the link between numbers and space has inspired a wealth of research (for recent review, see Fischer & Shaki, 2014). In the classic paradigm, participants make parity judgments (odd/even) of Arabic numerals using left and right response keys, with faster responses to smaller numbers when participants use the left key and faster responses to larger numbers when participants use the right key. This general finding has since been replicated using numerous paradigms. One such example is the magnitude comparison task in which participants indicate whether the digit shown is greater than or less than some central value (Dehaene, Dupoux, & Mehler, 1990). Additional work has demonstrated that simply perceiving numbers biases spatial attention: participants are faster to detect a leftward target when primed with a small digit and faster to detect a rightward target when primed with a large digit (Fischer, 2003; but see, e.g., Zanolie & Pecher, 2014, for replication failure). Further, changes in spatial attention bias number generation: for instance, when participants are asked to randomly generate numbers while making alternating left/right head movements, they more frequently generate small numbers when their head is oriented to the left and large numbers when their head is oriented to the right (Loetscher, Schwarz, Schubiger, & Brugger, 2008).

A common theory of the spatial-numerical associations (SNAs) described above is that they are the product of a stable mental number line (MNL), wherein smaller numbers are represented on one side of space and larger numbers are represented on the other (in Western cultures, smaller numbers represented on the left and larger numbers on the right; e.g., Dehaene et al., 1993). Yet there remain objections to this theory. Proctor and Cho (2006), for example, argued that polarity correspondence (a +/- categorization of stimulus and response) can explain the observed associations. Indeed, many tasks rely on a dichotomous response (e.g., left/right keys, left/right head position), and may be explained in this way. Another view, which has posed an even greater challenge to the MNL account, argues that SNAs are a product of task-specific associations established online within working memory (WM; van Dijck & Fias, 2011). The crux of this debate is whether the observed SNAs are driven from a stable spatial-numerical link (e.g., MNL) or by transient mappings of number onto space (e.g., polarity correspondence, or a WM account; for further discussion, see Cheung et al., 2015).

In general, those who argue in favor of a WM account argue that the ostensibly transient mappings are, at least in part, a product of task demands. For example, in the classic parity judgment task, participants respond using leftward and rightward oriented keys. One may argue that the

Table 1: SNA tasks and their task demands.

Category	**Examples**	**Task demands**			**Ordinal control**
		Dichotomous Categorization	**Directional prime**	**Explicit Magnitude**	
Parity Judgment	Dehaene et al., 1993 Marghetis et al., 2013	✓	✓	✓	✓
Mag. Judgment	Fitousi et al., 2009 Marghetis et al., 2013	✓	✓	✓	x
Lat. Comparison	Lavidor et al., 2004 Cheung et al., 2015	✓	✓	✓	x
Numerical Posner	Fischer et al., 2003 Ruiz Fernández et al., 2001	x	✓	x	x
Num. Bisection	Fischer et al., 2001 Calabria & Rossetti, 2005	x	✓	x	x
Number generation	Loetscher et al., 2008 Cheung et al., 2015	x	✓	✓	x
Eye-tracking	Holmes et al., 2016 Schwarz & Keus, 2004	x	x	✓	x
Place-the-number		x	x	x	✓

relative orientation of these keys is sufficient to induce a spatial mapping (see Viarouge, Hubbard, & Dehaene, 2014 for discussion of spatial reference frames involved in SNA tasks). In particular, a polarity correspondence account would be concerned about the use of a dichotomous response. Indeed, many of the SNA tasks mentioned above possess some kind of task demand. Parity judgment and magnitude comparison tasks involve both a dichotomous manual response (left key/right key) *and* a dichotomous judgment (odd/even or greater than/less than). Other parity and magnitude judgment tasks have utilized a go/no-go paradigm to circumvent the spatial information provided by the response keys, but these tasks nevertheless depend on a dichotomous response scheme (e.g., Marghetis, Kanwal, & Bergen, 2013). This dichotomous response, though not spatial, nevertheless lends itself to a polarity correspondence account. Further, even tasks that do not require a dichotomous response still have particular features that may instantiate a left-to-right reference frame. For example, in the work of Fischer and colleagues (2003), a left-to-right frame may be induced by the locations of the target (as either on the left or right side of fixation). The same may be said for the paradigm utilized by Ruiz Fernández and colleagues (2011), wherein, after presentation of a number, they made an arbitrary selection between items construed on the left and right sides of space. In the work of Loetscher and colleagues (2008), a left-to-right frame was specifically induced by the turning of the head. (For a more complete list of SNA paradigms and their task demands, see Table 1). Thus, it is unclear whether there is any evidence of an SNA (and consequently, a stable MNL) in the absence of any such task demands.

Eye-tracking paradigms have been promising in this regard. For instance, Holmes, Ayzenberg, and Lourenco (2016) had participants play a virtual blackjack game while their eye gaze was being tracked. It was found that both the value of a card on a given trial as well as the overall value of one's hand at a given time significantly predicted eye gaze in a manner consistent with observed SNAs for Western participants (i.e., smaller magnitudes produced more leftward eye movements and larger magnitudes produced rightward eye movements). This study provides strong evidence for a left-to-right oriented MNL by demonstrating that number representations bias spatial attention even in the absence of a directional prime and a dichotomous response scheme (see Table 1). However, this task required the explicit processing of numerical value (e.g., value of a card or hand). Even eye-tracking studies which have yielded similar results without requiring explicit magnitude processing (e.g., Schwarz & Keus, 2004; Loetscher et al., 2010) still invoke some property of number (e.g., parity). As such, two questions remain: do numbers *automatically* bias spatial attention in the absence of a directional prime and even when numerical properties are irrelevant to the task at hand? Furthermore, and critically, is this bias specific to number?

Automaticity as a criterion for representation

Understanding whether or not SNAs manifest automatically (i.e., in the absence of task demands) is crucial for understanding the relationship between numbers and space, in large part because automaticity suggests that the relationship is representational (as the MNL account predict) rather than transient (as polarity correspondence or WM accounts would predict). Nowhere is this criterion more apparent than in the literature on synesthesia. Automaticity, here, is where many have drawn the line between a relationship that is merely associative as opposed to truly synesthetic (see Grossenbacher & Lovelace, 2001; Mattingley, 2009). We argue that the spatial-numerical relationship should be considered in similar terms (see also, Cohen Kadosh & Henik, 2007). By this criterion, automaticity helps us to understand the nature of the relation between numbers and space: namely, whether they share representational resources or whether the two are only transiently associated with one another.

Here, we present evidence from a novel SNA paradigm—the place-the-number task—which suggests that numbers automatically bias spatial attention. Very simply, participants viewed a number on a screen, memorized its location and, after a delay, placed the number back in its original location. This task revealed a robust spatial-numerical relationship. In two additional control experiments, we found no consistent mapping of letters to space, suggesting that these attentional biases are specific to number and not ordinal sequences more generally. The comparison with other ordinal sequences has not always been tested with other paradigms. Here we emphasize that predictions made by MNL and WM accounts of SNAs diverge in such conditions, with an MNL account predicting spatial biases specific to number an a WM account predictings generalization across ordinal sequences (e.g., letters, months, etc.; van Dijck et al, 2014).

Experiment 1: Place-the-number task

Method

In this novel paradigm, participants viewed an Arabic numeral (1-9) presented in black font within a rectangle (white fill with black outline; 918 × 495 pixels). This task was created in Visual Basic and presented on a 19in computer monitor. Participants sat approximately 65cm from the monitor. Each digit was presented 20 times, for a total of 180 trials, randomly ordered. On each trial, participants were instructed to remember the location of the digit. Participants then clicked a button located at the bottom of the screen, at which time the digit disappeared. Participants were then instructed to click the remembered location in order to make the digit appear at that location. These instructions were presented in a pop-up dialog box, which also ensured that participants did not fixate on the original location of the digit. Participants could further adjust this initial placement by dragging and dropping the digit. Participants then confirmed their final placement by clicking another button on screen and immediately proceeded to the next trial.

Thirty-seven undergraduates participated in this task for course credit. All participants had normal or corrected-to normal vision. Procedures were approved by the Institutional Review Board (IRB). One participant was excluded from the statistical analyses due to poor accuracy (mean accuracy greater than 2.5 standard deviations (SD) from the mean group accuracy), where accuracy was calculated as the distance between the digit's original location and the participant's final placement.

Results

The remaining participants ($N = 36$) had a mean accuracy of 18.76 pixels (SD = 13.67). For each participant, trials with an accuracy greater than 2.5 SD from their individual mean accuracy were removed from subsequent analyses (mean number excluded per participant = 3.94). The variable of interest was participants' accuracy along the horizontal axis. For each trial, we calculated accuracy as the difference between the x-coordinate of the participant's final placement and x-coordinate of the digit's original location, such that a negative value represents a more leftward placement, in comparison to the original location, and a positive value represented a more rightward placement, in comparison to the original location. For each participant, we calculated the mean accuracy for each digit and calculated a slope by regressing these values onto their corresponding numerical value (see Fig. 1). Thus, in this paradigm, a positive slope represents the canonical, left-to-right SNA. Consistent with this, participants' slopes were significantly greater than zero, $t(35) = 2.69$, $p = .01$, $d = .45$. Furthermore, a significant number of participants ($N = 23$) showed this effect (binomial test, $p < .05$). In other words, participants placed smaller numbers more leftward than larger numbers, consistent with a left-to-right SNA. To determine whether the SNA shown here was unique to number (as an MNL account predicts) or occurs for any ordinal sequence (as a WM account predicts), we conducted a control experiment (Exp. 2A) with letters.

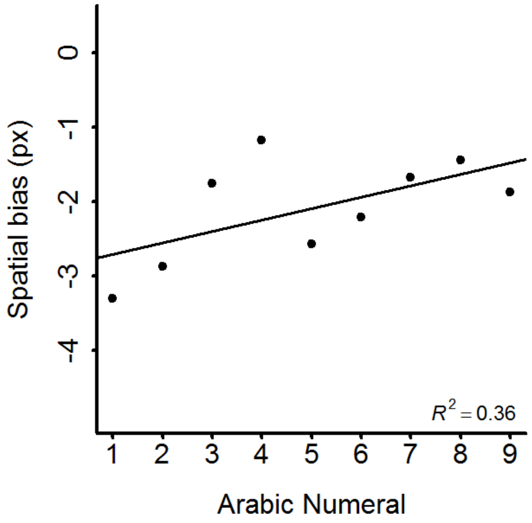

Figure 1: Scatterplot displaying mean spatial bias (final – original placement) for digits 1-9 including the best-fitting regression line.

Experiment 2A: Letter control (A-I)

Method

The procedure for Experiment 2A was identical to Experiment 1 except that instead of Arabic numerals as stimuli, participants were presented with the first nine letters of the alphabet (A-I). Thirty-eight undergraduates participated for course credit. One participant was excluded from the statistical analyses as they did not complete all trials. One participant was excluded from the statistical analyses for poor accuracy.

Results

The remaining participants (*N* = 36) had a mean accuracy of 13.76 pixels (SD = 6.75). For each participant, trials with an accuracy greater than 2.5 SD from their individual mean accuracy were removed from subsequent analyses (mean number of exclused trials per participant = 3.47). Importantly, unlike Experiment 1, participants' slopes were not significantly different from zero, *t*(35) = -1.23, *p* = .23. This result demonstrates that letters, although ordinal, did not generate a spatial association in this paradigm. However, since the letters used only spanned the beginning of the alphabet, it remains possible this sequence was not comparable to the Arabic numerals used in Experiment 1. Indeed, a WM account might predict that the default (canonical) set of letters that would be relevant to the task should apply to the entire sequence. The following experiment was designed to address this concern.

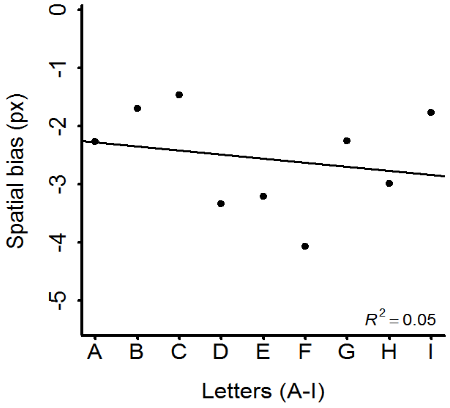

Figure 2: Scatterplot displaying mean spatial bias (final – original placement) for letters A-I including the best-fitting regression line.

Experiment 2B: Letter control (A-Z)

Method

The procedure for Experiment 2B was identical to Experiment 1 and 2A but instead of Arabic numerals as stimuli, nine letters evenly spaced throughout the alphabet (A, D, G, J, M, P, S, V, Y) were presented, as we hypothesized participants could more easily distinguish between the ordinal position of "A"/"Y" than "A"/"I", for example. Thirty-eight undergraduates participated for course credit. One participant was excluded from the statistical analyses as they did not complete all trials. One participant was excluded from the statistical analyses for poor accuracy

Results

The remaining participants (*N* = 36) had a mean accuracy of 13.65 pixels (SD = 9.11). For each participant, trials with an accuracy greater than 2.5 SD from their individual mean accuracy were removed from subsequent analyses (mean number of trials excluded per participant = 3.27). Unlike Experiment 1, and consistent with the findings of Experiment 2A, participants' slopes were not significantly different from zero, *t*(35) = 1.42, *p* = .16, confirming that letters do not generate a spatial association in this paradigm.

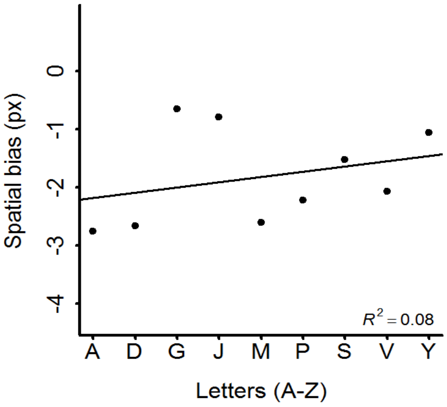

Figure 3: Scatterplot displaying mean spatial bias (final – original placement) for letters evenly distributed in the alphabet including the best-fitting regression line.

General Discussion

A primary goal of this study was to test for a SNA in Western adults in the absence of any sort of task demand. In the place-the-number task, participants' responses are non-dichotomous; there is no left-right directional prime; and the value of the stimulus is not necessary to complete the task. Yet participants nevertheless exhibited a left-to-right SNA. Our two control experiments demonstrate that this effect is specific to number, as opposed to all ordinal sequences such as letters.

Consistent with our hypotheses, these findings demonstrate that numbers—but not letters—*automatically* bias spatial attention in accordance with an MNL account. The fact that this effect occurs in the absence of relevant task demands is critical. Those who posit alternative accounts of SNAs often offer explanations that rest on demand characteristics of the tasks themselves, but there are no such demands here. Therefore, we conclude that the automaticity of the spatial bias in this task sheds light on the nature of numerical representations—that they are inherently spatial unlike other ordinal sequences. Though other accounts exist, we believe these results provide strong evidence in favor of an MNL account of SNAs.

This interpretation is consistent with recent neural work which has explored the relation between space and number (Harvey et al., 2013). Harvey et al. (2013) found evidence of a topographic map for numerosity in the posterior superior parietal lobule, akin to topographic maps for sensorimotor systems. Within this area, medial regions preferred small numerosities and lateral regions preferred large numerosities. Importantly, the location and numerosity preference of this topographic map was consistent across participants. These data provide unique support for the view that numerical representations have an *inherent* spatial organization.

Spatial representation versus spatial strategy

One question that arises from this interpretation is why others have demonstrated effects of ordinal sequences (e.g., Gevers, Reynovet, & Fias, 2003; van Dijck & Fias, 2011). To answer this question, we make a critical distinction. On the one hand, we might ask what things we *can* organize spatially; on the other hand, we might ask what things are *inherently* spatial. It is only in the context of the latter question that we argue numbers are unique. That we can organize ordinal sequences spatially should come as no surprise. People can organize items in any number of spatial arrangements and this type of spatialization has often been considered important for reasoning (Johnson-Laird, 1983). But the question that concerns the authors here is whether numbers are unique in the sense that they are automatically represented spatially in the mind.

This dichotomy is reminiscent of the so-called “dual-process” model of SNAs (e.g., Ginsburg & Gevers, 2015; see also, Cheung & Lourenco, 2016) which entails both long-term SNAs as well as spatialization in WM. Abrahamse, van Dijck, and Fias (2016) have argued against this view, suggesting that their WM account is more parsimonious—that it “captures the complexity of the empirical database” without the need for long-term associations (p. 7). Indeed, if the mind were constructed for the sole purpose of representing number, then it might have evolved to do so in a parsimonious manner. Yet, Abrahamse et al. (2016) ignore the possibility that multiple mechanisms, some of them domain-general, may be at play. As we suggest, all ordinal sequences can be represented in space, but only numbers are automatically represented in this manner.

A WM account of SNAs suggests that we have the propensity to organize sequences spatially in order to minimize the load of maintaining the sequence in the mind at once. This idea is reminiscent of the "method of loci" — a means of improving memory per spatial visualization — is at least two millennia old (as in Cicero’s *De Oratore*), but it has nothing to do with intrinsic characteristics of the representation. Thus, it becomes important to differentiate questions of spatial *representation* and spatial *strategy*. Previous tasks, some rife with potential task demands, failed to make a distinction between these two perspectives, yet have been interpreted as evidence of spatial *representation*. van Dijck and Fias (2011) have argued, partially on account of these task demands, that SNAs are merely transient mappings that occur in WM—which, within our framework, falls under the purview of spatial *strategy*. In pursuing the latter issue of strategy, those who have espoused this WM perspective have overlooked the former, more crucial question of representation. In other words, although we have argued that numbers are unique insofar as they are inherently spatial, van Dijck and Fias (2011) have succeeded only in showing that other sequences can, in certain contexts, be mapped spatially.

Empirical horizons

What does the distinction between spatial representation and spatial strategy buy us? As a starting point, it establishes that numbers are in fact unique: they bias spatial attention *automatically* which suggests that their representation is *inherently* spatial in a way that other ordinal sequences are not. With this in mind, we are able to ask more nuanced questions about the underlying relationship between numbers and space. For example, why are numbers unique in this way? Do we come into the world with the propensity to represent numbers spatially, or is it learned? Perhaps more critically: what is the utility of a spatial-numerical mapping? For example, despite the seeming ubiquity and permanence of SNAs, it is unclear whether this spatial-numerical bias is related to math performance, with some studies reporting a positive relationship, some a negative relationship, and some no relationship at all (for review, see Cipora, Patro, & Nuerk, 2015).

Not only does the place-the-number task play a part in raising these questions, it may also help to answer them. To further understand the phylogenetic and ontogenetic development of these associations, it is necessary to examine them in early childhood as number concepts are still being acquired. This has proven challenging, however, given that many SNA tasks are difficult to administer to children. The place-the-number task alleviates these concerns and might allow for the study of SNAs at a time in development when they have greater utility.

In sum, we have shown that numbers, but not letters, bias spatial attention in a manner that is consistent with an MNL hypothesis of SNAs. We argue that previous work which has posited alternative explanations to this account have been inadvertently answering a separate question—one about spatial strategy rather than spatial representation. Here, we have clarified the difference between these two accounts and suggested how the place-the-number task may be used to guide future research on the deep relationship between numbers and space in the mind.

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